Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 5, 1999

REFLECTION OF SHOCK WAVES FROM A SOLID BOUNDARY IN A MIXTURE OF CONDENSED MATERIALS. 1. EQUILIBRIUM APPROXIMATION

A. A. Zhilin and A. V. Fedorov

UDC 532.529

The process of reflection of shock waves (SW) from a solid wall in a two-component mixture of condensed materials is studied within the framework of mechanics of heterogeneous media. The velocity of a reflected SW and the values of the parameters behind its front are analytically determined as functions of the velocity of the incident wave and the initial parameters of the mixture. It is shown that the absolute value of the velocity of the reflected SW can be greater than the velocity of the incident SW in mixtures with a small content of the light component and at low velocities of the incident shock wave. The nonmonotonic character of the dependence of pressure in the final equilibrium state behind the incident SW on the initial volume concentration of particles is demonstrated. The velocity of the incident SW is estimated for the case where a similar effect is also observed behind a reflected SW. It is established that, for weak shock waves, the dependence of the amplification factor of the reflected SW on the initial volume concentration of the light component is nonmonotonic and has a local maximum. It is noted that, as the velocity of the incident SW increases, the effect of compacting of the mixture (increase in concentration of the heavy component) behind the reflected SW becomes much less pronounced than in a passing SW.

The problem of propagation and reflection of shock waves (SW) from a solid wall in mulitcomponent mixtures is of great theoretical and practical interest and has not been studied in detail yet. Fedorov [1] considered problems of the SW structure in a mixture of two condensed materials in the one-velocity approximation with different pressures of the components of the mixture, and also in a mixture with account of the difference in phase velocities and pressures in the case of an infinitely large relaxation time of the volume concentration of the heavy component of the mixture τ_{m_2} and a finite time of velocity relaxation. It is shown that the resultant dispersed and frozen SW structures are characterized by monotonically decreasing phase velocities with bow and/or internal shock waves. Fedorov and Fedorova [2] performed a numerical and analytical study of the problem of reflection of a shock wave from a solid boundary in a mixture of two compressible media under the condition $\tau_{m_2} \to \infty$. Thus, the study was conducted within the framework of the model of mechanics of heterogeneous media with different velocities and pressures of the components, and the volume concentration was $m_2 \equiv$ const. It was found by calculations that the SW retains its type, being reflected from the solid wall in such a mixture. It should be noted that the process of equalization of pressures of the components of the mixture behind the SW front, i.e., the finiteness of the relaxation time τ_{m_2}, was not taken into account in [2].

The mathematical model in the more general case of a finite relaxation time τ_{m_2} without constraints on the materials of the components of the mixture examined was studied in [3, 4]. Within the framework of the proposed approach, the existence of different SW structures was shown: fully dispersed, frozen-dispersed, dispersed-frozen, and frozen shock waves of two-wave configuration with a monotonically decreasing or nonmonotonic velocity profile. Zhilin and Fedorov [5] studied the process of stabilization of steady wave

Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 5, pp. 73–78, September–October, 1999. Original article submitted December 22, 1997.

0021-8944/99/4005-0841 $22.00

structures obtained in [3, 4] in a heterogeneous mixture. Their stability to finite and infinitesimal disturbances was also shown, which allowed the authors to solve the problem of SW initiation from stepwise initial data.

The goal of the present paper is to study the process of reflection of different SW types obtained in [3, 4] from a solid wall. This problem was considered previously in mechanics of heterogeneous media in the case of equal pressures of the components of the mixture. Thus, Miura et al. [6] studied flows resulting from SW interaction with a solid wall in a dust-laden mixture in the two-velocity and two-temperature approximation. In the equilibrium approximation, it was proved that there are three types of transition of incident SW (with a frozen and dispersed structure) to reflected SW depending on the pressure difference on the incident SW: 1) both incident and reflected waves are frozen; 2) a frozen incident SW is reflected as fully dispersed; both incident and reflected shock waves are characterized by a fully dispersed structure. The model considered adequately describes gas mixtures and does not take into account the difference in phase pressures; in addition, it is assumed that the change in the volume fraction of the particles can be ignored. In what follows, in contrast to [2], the problem of SW reflection is studied within the framework of the model that takes into account relaxation of phase pressures.

Physicomathematical Formulation of the Problem. We consider a mixture of two condensed materials consisting of a light component, which occupies continuously the entire volume, and a heavy component, which is discretely distributed in the light component. An SW propagates from right to left in a quiescent mixture with equilibrium initial parameters. Behind the SW front, after the relaxation zone, the parameters of the mixture correspond to the equilibrium final state in terms of velocities and pressures of the components. The left boundary is a solid wall ($x = 0$); being reflected from this wall, the SW forms a new equilibrium state in the medium behind the SW front. The problem is to determine the parameters of the components of the mixture in the region $x \geqslant 0$, $t \geqslant 0$.

The equations that describe the flow of the mixture in dimensionless variables have the form

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 u_1}{\partial x} = 0, \qquad \frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 u_2}{\partial x} = 0, \qquad \frac{\partial \rho_1 u_1}{\partial t} + \frac{\partial \rho_1 u_1^2}{\partial x} = -m_1 \frac{\partial P_1}{\partial x} + F_S,$$

$$\frac{\partial \rho_2 u_2}{\partial t} + \frac{\partial \rho_2 u_2^2}{\partial x} = -m_2 \frac{\partial P_2}{\partial x} - (P_2 - P_1) \frac{\partial m_2}{\partial x} - F_S, \qquad \frac{\partial m_2}{\partial t} + u_2 \frac{\partial m_2}{\partial x} = R, \tag{1}$$

$$m_1 = 1 - m_2, \qquad P_1 = \rho_1/m_1 - 1, \qquad P_2 = a^2(\rho_2/m_2 - \bar{\rho}),$$

where ρ_i, u_i, P_i, and m_i are the mean density, velocity, pressure, and volume concentration of the ith component of the mixture, $F_S = m_1\rho_2(u_2 - u_1)/\tau_S$ is the Stokes force, $\tau_S = 2\bar{\rho}/(9\mu_1)$ is the time of velocity relaxation under the action of the Stokes forces, $R = m_1 m_2(P_2 - P_1)/\tau_{m_2}$ is a function that describes the transfer of the solid phase, $\tau_{m_2} = 2\mu_2$ is the relaxation time of the volume concentration of the heavy component of the mixture, μ_i is the dynamic viscosity of the ith component, $a = a_2/a_1$, $\bar{\rho} = \rho_{22,0}/\rho_{11,0}$, $\rho_i = m_i\rho_{ii}$, ρ_{ii} is the true density of the ith component, and a_i and $\rho_{ii,0}$ are the speed of sound and the true density of the material of the ith component of the mixture. The velocities are normalized to a_1, the densities to $\rho_{11,0}$, the pressures to $a_1^2\rho_{11,0}$, the spatial coordinate x to the radius of the solid particles r, and the time t to $t_0 = r/a_1$.

We determined the equilibrium C_e and equilibrium-frozen $C_{e,f}$ speeds of sound in the mixture

$$C_e^2 = \frac{\xi_1}{m_1} \frac{m_1 C - \rho\xi_1}{m_1^2 C - \rho\xi_1}, \qquad C_{e,f}^2 = \xi_1 + a^2\xi_2,$$

where $C = 1 - a^2\bar{\rho}$ and $\xi_i = \rho_i/\rho$.

The initial-boundary conditions for Eqs. (1) can be presented in the following form:

$$\varphi = \varphi_0(x), \quad x \geqslant 0, \quad t = 0; \qquad u_1 = u_2 = 0, \quad x = 0, \quad t \geqslant 0. \tag{2}$$

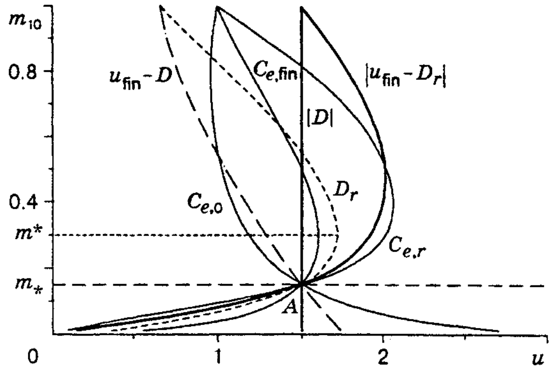

Fig. 1

The vector of the solution $\varphi_0(x)$ here describes a steady SW with one of the structures found in [3, 4]. Thus, the problem of SW reflection from a solid wall reduces to the solution of the initial-boundary problem (1), (2).

Calculation of the Parameters of a Reflected SW in the Equilibrium Approximation. The initial equilibrium state is characterized by the parameters $\rho_i = \rho_{i0}$, $u_i = 0$, and $P_i = 0$ upstream of the incident SW front. Behind the SW front, the parameters of the mixture take the final equilibrium values $\rho_i = \rho_{i,\rm fin}$, $u_i = u_{\rm fin}$, and $P_i = P_{\rm fin}$. The equilibrium state behind the front of the reflected SW is described by the parameters $\rho_i = \rho_{ir}$, $u_i = u_r = 0$, and $P_i = P_r$. From the conservation laws of mass and momentum for the incident and reflected SW, we have

$$\begin{gathered} -\rho_0 D = \rho_{\rm fin}(u_{\rm fin} - D), \quad \rho_0 D^2 = P_{\rm fin} + \rho_{\rm fin}(u_{\rm fin} - D)^2, \\ -\rho_r D_r = \rho_{\rm fin}(u_{\rm fin} - D_r), \quad P_r + \rho_r D_r^2 = P_{\rm fin} + \rho_{\rm fin}(u_{\rm fin} - D_r)^2, \end{gathered} \tag{3}$$

where D_r and $D_r = D$ are the velocities of the incident and reflected shock waves, respectively.

For determining D_r, after some transformations, we obtain the cubic equation

$$\begin{gathered} D_r^3 u_{\rm fin}^2 \rho_{\rm fin}^2 - u_{\rm fin}\rho_{\rm fin}(2 - C + 2u_{\rm fin}\rho_{\rm fin} D - C_{e,f}^2 \rho_{\rm fin}) D_r^2 - [C - 1 + \rho_{\rm fin}(C_{e,f}^2 - C\xi_1) \\ -u_{\rm fin}\rho_{\rm fin} D(2 - C) - u_{\rm fin}^2 \rho_{\rm fin}^2 (D^2 - C_{e,f}^2) + C_{e,f}^2 u_{\rm fin}\rho_{\rm fin}^2 D] D_r + \rho_{\rm fin} u_{\rm fin}(C_{e,f}^2 - C\xi_1 + u_{\rm fin}\rho_{\rm fin} D C_{e,f}^2) = 0, \end{gathered}$$

which has a trivial solution $D_r = D$ corresponding to the incident SW velocity. The remaining quadratic equation

$$D_r^2 u_{\rm fin}^2 \rho_{\rm fin}^2 - D_r u_{\rm fin}\rho_{\rm fin}(2 - C + u_{\rm fin}\rho_{\rm fin} D - C_{e,f}^2 \rho_{\rm fin}) - C + 1 - \rho_{\rm fin}(C_{e,f}^2 - C\xi_1) - u_{\rm fin}^2 \rho_{\rm fin}^2 C_{e,f}^2 = 0$$

has two roots: $D_r^{\pm} = [2 - C + u_{\rm fin}\rho_{\rm fin} D - C_{e,f}^2 \rho_{\rm fin} \pm \sqrt{\mathcal{D}}]/(2u_{\rm fin}\rho_{\rm fin})$. The radicand $\mathcal{D} = (C - u_{\rm fin}\rho_{\rm fin} D + C_{e,f}^2 \rho_{\rm fin})^2 + 4\rho_{\rm fin}(u_{\rm fin}^2 \rho_{\rm fin} C_{e,f}^2 - C\xi_1 + u_{\rm fin} D)$ is always positive, since $C < 0$ and $u_{\rm fin} D > 0$. Hence, two real values of the velocity of the reflected SW can exist. Physically meaningful is the lower branch where $D_r^- > 0$ (it corresponds to the sign minus, since $u_{\rm fin} < 0$). The solutions D_r^+ on the upper branch are negative. In what follows, the sign minus in D_r^- is omitted: $D_r = D_r^-$. Figure 1 shows the behavior of the velocity of the reflected SW (dashed curve D_r), relative phase velocities behind the incident SW front (dot-and-dashed curve $u_{\rm fin} - D$) and ahead of the reflected SW (heavy solid curve $|u_{\rm fin} - D_r|$), and equilibrium speeds of sound (thin solid curves $C_{e,0}$, $C_{e,\rm fin}$, and $C_{e,r}$ in the initial and final equilibrium states both behind the incident and reflected shock waves) versus m_{10} (the incident SW velocity is $D = -1.5$).

In Fig. 1, we can see a region of unstable flow (located below the assembly point A of the characteristic flow velocities), in which the Zemplén theorem is invalid. In the region above this point, the conditions of the Zemplén theorem are fulfilled both for the incident and reflected SW: $|D| > C_{e,0}$, $|u_{\rm fin} - D| < C_{e,\rm fin}$,

TABLE 1

Equilibrium Parameters of the Mixture behind the Incident and Reflected SW

m_{10}	$u_{\rm fin}$	$m_{1\rm fin}$	$P_{\rm fin}$	D_r	m_{1r}	P_r	$\lvert u_{\rm fin} - D_r\rvert$	k
				$D = -1.5$				
0.15	−0.001	0.1495	0.0043	1.503	0.1490	0.0086	1.504	2.003
0.20	−0.076	0.167	0.263	1.636	0.140	0.578	1.711	2.201
0.30	−0.213	0.207	0.688	1.714	0.145	1.719	1.927	2.497
0.40	−0.338	0.257	1.008	1.671	0.165	2.750	2.009	2.728
0.50	−0.452	0.320	1.236	1.566	0.200	3.615	2.018	2.924
0.60	−0.555	0.400	1.382	1.421	0.251	4.272	1.976	3.091
0.70	−0.648	0.502	1.4524	1.247	0.329	4.681	1.895	3.223
0.80	−0.728	0.634	1.4520	1.053	0.453	4.801	1.781	3.306
0.85	−0.762	0.712	1.426	0.952	0.542	4.740	1.714	3.324
0.90	−0.792	0.800	1.384	0.852	0.657	4.597	1.644	3.322
0.95	−0.816	0.896	1.325	0.755	0.807	4.370	1.571	3.298
				$D = -2.5$				
0.05	−0.166	0.026	1.067	2.681	0.017	2.369	2.847	2.220
0.10	−0.375	0.035	2.332	2.622	0.020	5.622	2.997	2.411
0.20	−0.685	0.055	3.974	2.446	0.030	10.828	3.131	2.725
0.30	−0.928	0.080	5.002	2.271	0.042	15.186	3.200	3.036
0.40	−1.138	0.110	5.659	2.095	0.056	19.084	3.232	3.372
0.50	−1.326	0.151	6.048	1.909	0.076	22.700	3.234	3.754
0.60	−1.499	0.208	6.222	1.704	0.104	26.135	3.203	4.201
0.70	−1.662	0.290	6.213	1.470	0.146	29.449	3.132	4.740
0.80	−1.817	0.416	6.042	1.191	0.220	32.655	3.008	5.405
0.90	−1.964	0.624	5.719	0.846	0.380	35.662	2.809	6.236

$|u_{\rm fin} - D_r| > C_{e,\rm fin}$, and $|D_r| < C_{e,r}$ for all $m_{10} > m_*$. The coordinate of the assembly point A for $D = -1.5$ corresponds to $m_{10} = 0.15$ and for $D = -2.5$ to $m_{10} = 0.02$.

In Fig. 1, we can also see the effect of nonmonotonic behavior of the reflected SW velocity depending on the ratio of volume concentrations of the components of the mixture. First, at $m_{10} > m_*$, an increase in the velocity of the reflected SW is observed with increasing fraction of the light component. The maximum value of the reflected SW velocity $D_{r,\rm max} = 1.714$ for $D = -1.5$ is reached at $m^* = 0.30$, and, for example, for $D = -2.5$, $D_{r,\rm max} = 2.681$ is reached at $m^* = 0.05$. For $m_{10} > m^*$, the velocity of the reflected SW decreases with increasing m_{10}. Numerical values of the reflected SW velocity D_r for different velocities of the incident shock wave D depending on the initial volume concentration of the light component of the mixture are presented in Table 1 for $D = -1.5$ and $D = -2.5$. Table 1 contains also the equilibrium parameters of the mixture $u_{\rm fin}$, $m_{1\rm fin}$, and $P_{\rm fin}$ behind the incident SW and m_{1r} and P_r behind the reflected SW. Note that there is no unstable flow region for $D = -3.3$, since the line of the equilibrium speed of sound C_e, which separates the regions of stable and unstable flows, is outside the limits of the considered interval of volume concentrations $m_{10} \in [0, 1]$ for the range of velocities of the incident SW $|D| > a$. In addition, the effect of nonmonotonic behavior of the reflected SW velocity as a function of m_{10} is not observed in this range of velocities and volume concentrations.

Note that the pressure behind the reflected SW front $P_{\rm fin}$ behaves nonmonotonically with increasing m_{10}. The nature of the maximum arising in $P_{\rm fin}$ is explained using the equation of state for the mixture in the equilibrium state behind the incident SW front

$$P_{\rm fin} = \frac{\rho_{1\rm fin}}{m_{1\rm fin}} - 1 = \frac{m_{10} D}{m_{1\rm fin}(D - u_{\rm fin})} - 1. \tag{4}$$

Some numerical values of $m_{1\rm fin}$ and $u_{\rm fin}$ versus m_{10} are listed in Table 1. The nonlinear dependence $P_{\rm fin}$ on

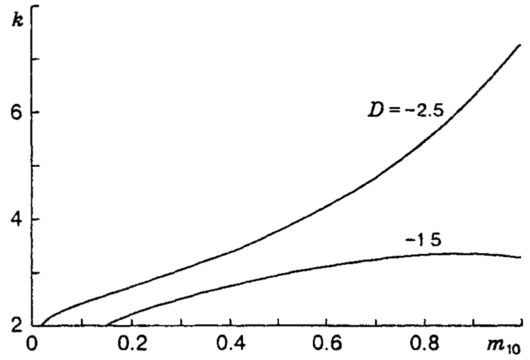

Fig. 2

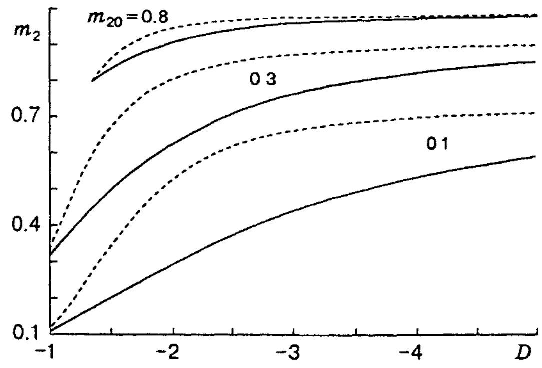

Fig. 3

m_{10} is the condition for existence of the pressure maximum $P_{\rm fin}^{\rm max}$ in the final equilibrium state behind the incident SW front. As the velocity of the incident SW increases, the position of $P_{\rm fin}^{\rm max}$ shifts toward the range of low values of m_{10} ($m_{10} = 0.75$ for $D = -1.5$, $m_{10} = 0.64$ for $D = -2.5$, and $m_{10} = 0.50$ for $D = -3.3$) and takes the values $P_{\rm fin}^{\rm max} = 1.461$, 6.239, and 12.654, respectively.

The equilibrium pressure behind the reflected SW front P_r increases with increasing m_{10}, reaches the maximum value $P_r^{\rm max} = 4.803$ at $m_{10} = 0.79$ for $D = -1.5$, and decreases on the remaining interval of m_{10}. The reason for the appearance of nonmonotonicity of the function $P_r = P_r(m_{10})$ is the same as in the above-considered case $P_{\rm fin} = P_{\rm fin}(m_{10})$. As the incident SW velocity increases, the maximum in P_r shifts toward greater m_{10}; for $D = -2.5$, the position of the maximum is outside the limits of the considered range of volume concentrations of the components of the mixture.

It follows from Table 1 and Fig. 2 that the amplification factor of the reflected shock wave $k = (P_r - P_0)/(P_{\rm fin} - P_0)$ with increasing m_{10} is characterized by a nonmonotonic behavior with a local maximum $k^{\rm max}$, which is reached at $D = -1.5$ for $m_{10} = 0.87$. Note that this maximum is a consequence of nonmonotonic pressures in the final equilibrium states both behind the incident and reflected shock waves. An increase in the velocity of the incident shock wave shifts the maximum of the amplification factor toward greater volume concentrations. The critical situation in terms of $k^{\rm max}$ is observed at a certain D and $m_{10} \approx 1$, after which a further increase in D, for example, to $D = -2.5$ (see Table 1 and Fig. 2), makes the function k monotonic over the entire interval of m_{10} from 0 to 1.

In applications, it is often important to evaluate the level of compacting of a two-component mixture under the action of an SW. We determined the volume concentration of the heavy component of the mixture behind the incident SW $m_{2\rm fin}$ (solid curves in Fig. 3) and reflected SW m_{2r} (dashed curves) as a function of the incident SW velocity D for different m_{20}. As the incident SW velocity increases, the volume concentration of the heavy component dramatically increases downstream both behind the incident and reflected shock waves, i.e., the light material is displaced and the heavy material is compacted in both cases. We note that there are two states with $m_{2\rm fin} = m_{2r}$. The first equilibrium point is located on the line of the equilibrium speed of sound C_e (point A in Fig. 1) if the SW propagates with the speed of sound. The second equilibrium state in terms of volume concentrations behind the incident and reflected shock waves is reached asymptotically as $D \to -\infty$, as seen from Fig. 3. The second equilibrium point m_2^* takes the values $m_2^* = 0.990$ at $m_{20} = 0.8$, $m_2^* = 0.911$ at $m_{20} = 0.3$, and $m_2^* = 0.726$ at $m_{20} = 0.1$. The greatest difference in volume concentrations of the components behind the incident and reflected SW is reached in the region adjacent to the first equilibrium point (for example, at $m_{20} = 0.8$, 0.3, and 0.1 for $D = -1.792$, -1.744, and -2.336, respectively).

Thus, we have obtained analytical asymptotic estimates of the parameters of the reflected SW in a mixture of two condensed materials, which show that, depending on the fraction of the first (second) component, the absolute value of the reflected SW velocity can be greater or smaller than the incident SW velocity.

REFERENCES

1. A. V. Fedorov, "Shock-wave structure in a mixture of two solids (a hydrodynamic approximation)," *Model. Mekh.*, **5(22)**, No. 4, 135–158 (1991).
2. A. V. Fedorov and N. N. Fedorova, "Structure, propagation, and reflection of shock waves in a mixture of solids (a hydrodynamic approximation)," *Prikl. Mekh. Tekh. Fiz.*, **33**, No. 4, 10–18 (1992).
3. A. A. Zhilin, A. V. Fedorov, and V. M. Fomin, "Travelling wave in a two-velocity mixture of compressible media with different pressures," *Dokl. Ross. Akad. Nauk*, **350**, No. 2, 201–205 (1996).
4. A. A. Zhilin and A. V. Fedorov, "Shock-wave structure in a two-velocity mixture of compressible media with different pressures," *Prikl. Mekh. Tekh. Fiz.*, **39**, No. 2, 10–19 (1998).
5. A. A. Zhilin and A. V. Fedorov, "Propagation of shock waves in a two-phase mixture with different pressures of the components," *Prikl. Mekh. Tekh. Fiz.*, **40**, No. 1, 55–63 (1999).
6. H. Miura, T. Saito, and I. I. Glass, "Shock-wave reflection from a rigid wall in a dusty gas," *Proc. Roy. Soc. London*, **A404**, 55–67 (1986).